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Evaluation of fibre strength characteristics on the basis of the fibre fragmentation test

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A proper description of the fibre fragmentation test becomes more and more important because of the continual development of new types of fibres, an example being the oxide composite fibres tested in the present work. An attempt at a strict formulation of the problem involved in the description of the fibre fragmentation test is presented. It takes into account a distortion of fibre strength distribution function caused by the fibre breakage process occurring under increasing external load. The fibre strength distribution function is determined by a distribution function of defects on a fibre as well as the distribution function of the defect strength. The formulation of the problem considered results in a system of differential equations which is solved assuming some simplifications. The solution leads to a choice of a particular set of statistical fibre strength characteristics. The procedure is applied to oxide composite fibres developed for a high temperature use.

1. Introduction

Real use of a composite material demands a large number of properties to be evaluated. A preliminary evaluation of such mechanical properties as tensile strength, fracture toughness, creep and creep strength, which depend upon composite parameters like fibre volume fraction, interface strength, etc., can now be done by calculation. It is based on both reliable mechanical models and proper input parameters of the constituent materials. Results of the calculations are to be used for the optimization of the composite structure. Finally, composite specimens with an optimal structure have to pass the necessary mechanical testing.

The most important input parameters are strength characteristics of fibres. The usual procedure for evaluating fibre strength is either a direct measurement of the tensile strength of fibres of various length or obtaining the strength scatter of fibres of a constant length and then determining a scale dependence of the fibre strength, assuming the validity of the Weibull distribution for the fibre strength [1, 2]. Such approaches have a disadvantage when using the results to estimate the composite strength. Namely, a scale dependence obtained by such a method does not necessarily coincide with that to be used in a corresponding failure model, because the appropriate dependence is influenced by removing fibre defects, occurring in a definite vicinity of the fibre break, from the active defect population.

No doubt the best way to determine strength characteristics of fibres to be used as a reinforcement in composites is to test a composite sample. The procedure might be very similar to that usually called "fibre fragmentation test" (see, for example, [3]), except that the latter is used, as a rule, for the evaluation of the fibre/matrix interface strength. But in fact, there is no real necessity to know fibre characteristics to evaluate the interface strength, and vice versa. It is obviously possible to construct a variety of models containing both sets of characteristics to be evaluated.

Curtin [4] developed a theory of fibre fragmentation in a single filament composite. He took into account a decrease of the effective fibre length equal to the original length minus the sum of so called recovery lengths around the fibre breaks. Then the solution of a set of differential equations describing kinetics of the accumulation of further fibre breaks was obtained, assuming the strength distribution for a decreasing effective fibre length to be unchanged. This assumption certainly leads to a decrease of exactness of the theory and the difference between the solution obtained and the real situation remains unknown. It should be noted that the author [4] compared his calculation with the results of a simulation of the fibre fragmentation process based on the Monte Carlo procedure performed previously by Henstenburg and Phoenix [5].

Mileiko and Glushko [6] evaluated the dependence of fibre strength upon the average free fibre length remaining after exclusion of the sum of recovery lengths surrounding the points of fibre breaks. In this case neither fibre strength distribution nor fibre defects distribution were considered.

In the present paper a new theory of the fibre fragmentation test is developed and used to obtain strength characteristics of fibres tested in an experiment of the fibre fragmentation type. Unlike the model developed by Mileiko and Glushko [6], a certain fibre defect distribution is an essential point of the model, and unlike a theory evaluated by Curtin [4], the defect distribution, and therefore the strength distribution, of the fibre is changing continuously with removal of the defects caused by fibre breakage. The problem formulated in a strict enough way does not look a trivial one, moreover it can have a non single solution.

The experiments have been carried out on a new type of oxide composite fibres which can be used as reinforcement at temperature as high as 1100–1200 °C [6, 7].

2. A model of fibre breakage

2.1. Fibre stress distribution

Let us consider in the short term the axial stress distribution in a fibre around a break point, within the framework of the unidirectional approach. Let r_f be the fibre radius, and the matrix occupies infinite volume in the z-axis direction, $r_f < r < R$. Away from a point of the fibre break, stresses p and σ_m in the fibre and matrix, respectively. are given by a solution of the system of equations:

$$\begin{aligned} p &= E_f \varepsilon \\ \sigma_m &= \sigma_m(\varepsilon) \\ Q &= p\pi r_f^2 + \sigma_m \pi (R^2 - r_f^2) \end{aligned} \tag{1}$$

Here Q is the axial load applied to a single filament composite and ε is the axial strain, which is supposed to be the same for the matrix and fibre. The stress–strain curve of the matrix material, $\sigma_m(\varepsilon)$, is assumed to be a known function. Excluding ε from Equation 1 yields a value of p, which is a function of specimen geometry, applied load, Q, and elasto–plastic properties of the components.

At the fibre break point $z = 0$, we have fibre stress $\sigma = 0$ and stress recovery curve $\sigma(z)$ depends strongly on elastic–plastic behaviour of the matrix and fibre/matrix interface properties. To deal with the situation we shall follow a well known scheme suggested by Kelly and Tyson [8]. Namely, the shear stress distribution on the fibre/matrix interface is approximated by piecewise function $\tau(z)$, $\tau = \pm\tau^*$ on length l, $\tau = 0$ outside the recovery length.

Then the conditions of the mechanical equilibrium yield

$$\sigma(z) = \begin{cases} \tau^* \dfrac{2z}{r_f} & \text{at } 0 \leqslant z \leqslant l(p) \\ p(Q) & \text{at } z > l(p) \end{cases} \tag{2}$$

Length, $2l(p)$, of the stress recovery zone is defined as

$$l(p) = \frac{r_f p}{2\tau^*} \tag{3}$$

This length grows monotonically when applied stress p increases. But the value of axial stresses $\sigma(z)$ within the recovery length $|z| < l(p)$ does not change with an increase of stress p in the fibre. Therefore, no other breaks may occur within length $2l(p)$ around an existing break.

Obviously, it is possible to use a more sophisticated shear stress profile, that will lead to a different fibre stress profile. The subsequent analysis will be valid provided the stress distribution satisfies two restrictions. First, the stress recovery length should not decrease with increasing of p. Secondly, the fibre stresses at a particular point within the stress recovery length should not increase with increasing p. Note, in the case of a purely elastic system with a perfect interface it is impossible to satisfy the second condition, so for such a system an alternative approach has to be developed.

2.2. Multiply fibre breakage

The fibre carries a set of defects statistically distributed along it. Fibre stress p sufficient to cause a defect to transform into a fibre break is to be called the defect strength. Now we shall introduce a strength distribution function for a set of the defects, as well as a distribution function of defect points characterized by mean number, λ, of defects per unit length.

When the axial fibre stress is p, all defects of strength $\sigma < p$ are located within the recovery lengths and consequently may not transform into fibre breaks. Hence, the rest of the defects, which may transform into breaks, are located outside the recovery length and have strength greater than p. Therefore, with increasing stress p, the distortion of the strength distribution function N (σ) will take place for a set of defects located outside the recovery lengths. Let the current strength distribution function be denoted $N(\sigma|p)$, that is the probability for a defect to have strength $\sigma^* < \sigma$, while fibre stress is equal to p. We can write

$$N(\sigma|p) = \begin{cases} N(\sigma) - N(p)/1 - N(p), & \sigma > p \\ 0, & \sigma \leqslant p \end{cases} \tag{4}$$

Note that in Curtin's theory [4] the distortion of the strength distribution function was not taken into account.

We now consider a single filament composite with total fibre length $L \gg l(p)$. Let the number of current fibre breaks be sufficiently large, $k(p) \gg 1$. Obviously, the value of k is physically a discrete one, but to make the analysis simpler we will treat k as a continuous function of p. Let the total average length of the recovery zones be $l_E(p)$. Fibre stress, p, is the external parameter. If a distance between neighbour breaks is larger than $2l(p)$, then the value of $l_E(p)$ can be easily determined, $l_E(p) = 2k(p)l(p)$. But due to the stochastic nature of defect distribution this is true for low values of p only, for a general case we have

$$\begin{cases} l_E(p) \leqslant 2kl(p) \\ l_E(p) \leqslant L \end{cases}$$

If fibre stress, p, gains increment, dp, then the number of the breaks increases by value dk. This increment is proportional to the density of defects which can be broken, λ, and to the remaining effective fibre length $L - l_E(p)$, i.e.

$$dk = \lambda(L - l_E(p))\{N(p + dp|p) - N(p|p)\} \tag{5}$$

The expression in the curly brackets defines the probability for a defect to have strength $\sigma \in [p, p + dp]$. Taking into account Equation 4, we can rewrite the last equation as

$$k'(p) = \lambda(L - l_{\mathrm{E}}(p))\frac{\mathrm{N}'(p)}{1 - \mathrm{N}(p)} \tag{6}$$

where $\mathrm{N}(p)$ is the original strength distribution function. Equation 6 accounts for new fibre breaks to arise outside the recovery length only. This equation describes the fibre breakage process provided function $l_{\mathrm{E}}(p)$ is known.

Let the recovery zones be combined in $n(k)$ "islands", ($n \leqslant k$). The average distance between neighbouring islands can be written as

$$\bar{d} = \frac{L - l_{\mathrm{E}}(p)}{n} \tag{7}$$

When a new fibre break occurs, one of three events can follow. First, a new island arises, and $n(k+1) = n(k) + 1$. That is obviously the case when the new recovery length does not intersect with a previous one. Second, the new recovery length intersects with a previous zone, then the number of islands remains unchanged, $n(k+1) = n(k) + 0$. Third, a new recovery length intersects with two previous ones, then the number of islands decreases, $n(k+1) = n(k) - 1$. That means

$$n(k+1) - n(k) = \delta_{\mathrm{n}} \tag{8}$$

where δ_{n} takes one of the values, $+1, 0, -1$. The corresponding probabilities are $\mathscr{P}_{+1}$, $\mathscr{P}_0$ and $\mathscr{P}_{-1}$, respectively.

Assuming function $n(k)$ to be continuous, Equation 8 written in the finite difference form can be approximated by the ordinary differential equation

$$\frac{\mathrm{d}}{\mathrm{d}k}n(k) \cong \bar{\delta}_{\mathrm{n}} = \mathscr{P}_{+1} - \mathscr{P}_{-1} \tag{9}$$

Here the bar over a symbol means averaging. So we have written the equation to describe the evolution of the islands configuration when k is increasing.

Strictly speaking, an island size increase should be taken into account with increasing value of p. That brings the possibility of the joining of two islands between two breaks events, but it can be shown that these effects play a secondary role. So we have

$$\frac{\mathrm{d}n}{\mathrm{d}p} = k'(p)(\mathscr{P}_{+1} - \mathscr{P}_{-1}) \tag{10}$$

Probabilities $\mathscr{P}_{+1}$, $\mathscr{P}_0$ and $\mathscr{P}_{-1}$ can be easily obtained if the distribution function of the distances between neighbouring islands, $\psi(x)$, is known. The average distance between islands defined by Equation 7 can be written as

$$\bar{d} = \int_0^{\infty} x\psi'(x)\,\mathrm{d}x \tag{11}$$

Using Equation 11 and assuming that a new fibre break can arise with equal probability at any point of the fibre outside the recovery length, we obtain

$$\begin{aligned} \bar{d}\mathscr{P}_{-1} &= \int_0^{l(p)} x\psi'(x)\,\mathrm{d}x + \int_{l(p)}^{2l(p)} (2l(p) - x)\psi'(x)\,\mathrm{d}x \\ \bar{d}\mathscr{P}_0 &= \int_{l(p)}^{2l(p)} (x - l(p))\psi'(x)\,\mathrm{d}x \\ &\quad + \int_{2l(p)}^{\infty} 2l(p)\psi'(x)\,\mathrm{d}x \\ \bar{d}\mathscr{P}_{+1} &= \int_{2l(p)}^{1} (x - 2l(p))\psi'(x)\,\mathrm{d}x \end{aligned} \tag{12}$$

Now we calculate the rate of change of the total recovery length with increasing fibre stress, p. A contribution to the value of the change of the sizes of the islands at a constant number of fibre breaks will be

$$\left.\frac{\partial l_{\mathrm{E}}}{\partial p}\right|_{k=\mathrm{const}} = 2n(p)l'(p) \tag{13}$$

The contribution of a new fibre break can be written as

$$\left.\frac{\partial l_{\mathrm{E}}}{\partial k}\right|_{p=\mathrm{const}} \cong \bar{\delta}_l \tag{14}$$

$$\delta_l = \begin{cases} x & \text{at } x < l(p) \\ l + (x - l)\dfrac{l}{x} & \text{at } x \geqslant l(p) \end{cases}$$

Here x is the distance between neighbouring islands. Averaging the value of δ_l over all probable values of x, we obtain

$$\bar{\delta}_l = \int_0^{l(p)} x\psi'(x)\mathrm{d}x + \int_{l(p)}^{\infty}\left(2l(p) - \frac{l^2(p)}{x}\right)\psi'(x)\,\mathrm{d}x \tag{15}$$

Note that

$$dl_{\mathrm{E}}(p) = \frac{\partial l_{\mathrm{E}}(p)}{\partial p}dp + \frac{\partial l_{\mathrm{E}}dk}{\partial k\,dp}dp$$

Therefore, Equations 13 and 14 yield

$$\frac{\mathrm{d}l_{\mathrm{E}}(p)}{\mathrm{d}p} = 2n(p)l'(p) + k'(p)\bar{\delta}_l \tag{16}$$

Now, setting initial conditions, corresponding to a lower value of defect strength $p_{\min}$, for the system of ordinary differential equations as

$$k(p_{\min}) = n(p_{\min}) = 1, \quad l_{\mathrm{E}}(p_{\min}) = 2l(p_{\min}) \tag{17}$$

we can find a solution to the system given by Equations 6, 10 and 16 and describe the process of fibre breakage.

2.3. An exact solution for low fibre stresses

A solution of the system given by Equations 6, 10 and 16 can be easily obtained for the case of low fibre stresses, i.e.

$$0 < p - p_{\min} \ll p_{\max} - p_{\min}$$

It is a set of asymptotic expansions of the expression for the corresponding parameters

$$n(p) = k(p)$$
$$l_E(p) = 2l(p)k(p)$$

and

$$k(p) = -\lambda L \times \ln(1 - N(p))$$

Here $N(p_{min}) = 0$ is assumed, and p_{max} is the maximum value of p corresponding to $l_E(p_{max}) = L$ and $n(p_{max}) = 1$.

2.4. Singular approximation

To proceed with a numerical solution of the system of Equations 6, 10 and 16 we need to evaluate distribution function, $\psi(x)$ [see Equation 11]. To simplify the evaluation let us approximate function $\psi'(x)$ by Dirak's function,

$$\psi'(x) = \delta(x - \bar{d}) \quad (18)$$

where average distance between neighbour islands $\bar{d}$ is defined by Equation 7.

Then Equations 9, 12 and 15 yield

$$\bar{\delta}_n = \begin{cases} 1 - \dfrac{2l}{\bar{d}} & \text{at } \bar{d} \geq l(p) \\ -1 & \text{at } \bar{d} < l(p) \end{cases} \quad (19)$$

$$\bar{\delta}_l = \begin{cases} 2l - \dfrac{l^2}{\bar{d}} & \text{at } \bar{d} \geq l(p) \\ \bar{d} & \text{at } \bar{d} < l(p) \end{cases} \quad (20)$$

For the distribution function of the defect strength we shall use a Weibull type function,

$$N(\sigma) = \left(\frac{\sigma - p_{min}}{p_{max} - p_{min}}\right)^{\beta} \quad (21)$$

Hence the average strength of the defects will be

$$\bar{\sigma} = \frac{\beta p_{max} + p_{min}}{\beta + 1}$$

The system of Equations 6, 10 and 16 with initial conditions given by Equation 17 has been solved numerically. Dependencies of the average number of islands upon the number of fibre breaks are shown in Fig. 1. The increase in the total length of the recovery zones, l_E/L, with the applied fibre stress is presented in Fig. 2.

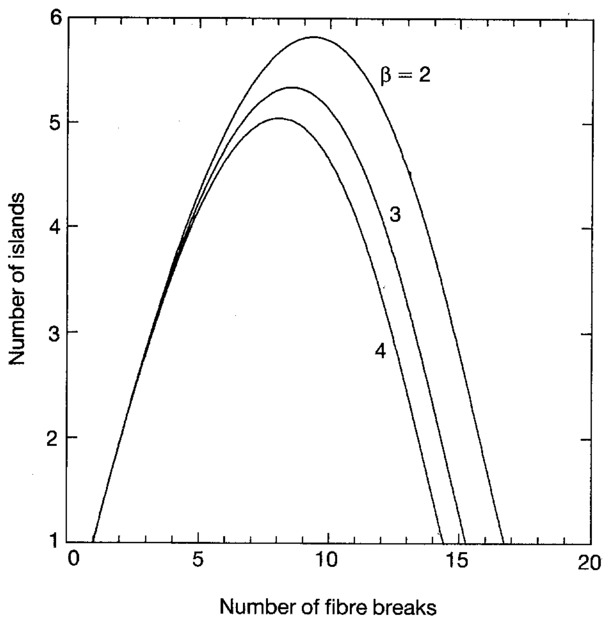

Figure 1 The average number of islands of the recovery zones versus the number of fibre breaks. Composite parameters are $\tau^* = 40$ MPa, $\bar{\sigma} = 1000$ MPa, $p_{min} = 500$ MPa, $\lambda = 4\,mm^{-1}$, $L = 35$ mm, $r_f = 0.18$ mm.

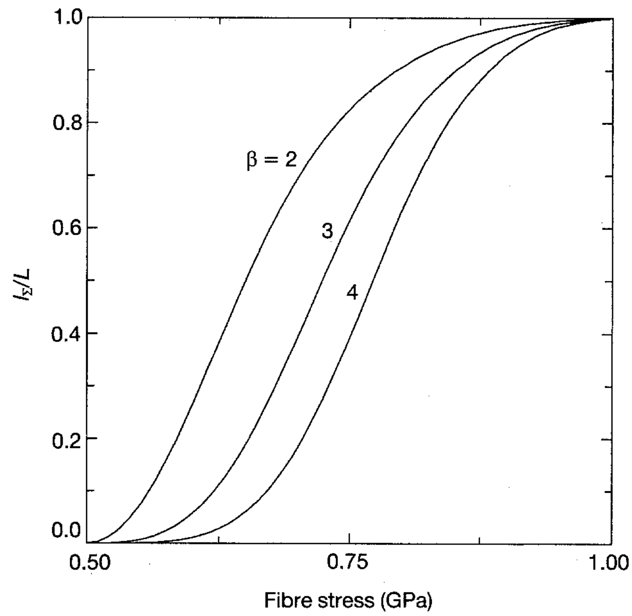

Figure 2 The total length of recovery zones l_E normalized by original fibre length L versus the applied fibre stress. Composite parameters are the same as those in Fig. 1.

Such curves contain all the necessary data to compare the results of the physical experiment with computer simulation data to obtain the fibre strength characteristics.

3. Experimental procedure

All the specimens were of an oxide composite fibre/copper matrix composite obtained by diffusion bonding at a temperature of 600 °C, a pressure of 80 MPa and a time of 30 min. Fibre diameter was 0.38 mm, the length of a specimen and the fibre length 35 mm. The specimen thickness was about 0.5 mm.

Tensile tests were carried out using a sufficiently rigid machine to record load drops through a normal load cell bringing its signal to the Y input of a X–Y recorder. An example of the original curve is shown in Fig. 3. The stress–strain curve of the pure matrix is presented in Fig. 4.

4. Fibre strength characteristics

The load applied to a specimen, Q_k, which caused k-break of the fibre was registered by the X–Y recorder, and a corresponding value of fibre stress, p_k, was calculated using Equation 1. The results obtained in testing three specimens are presented in Figs 5–7.

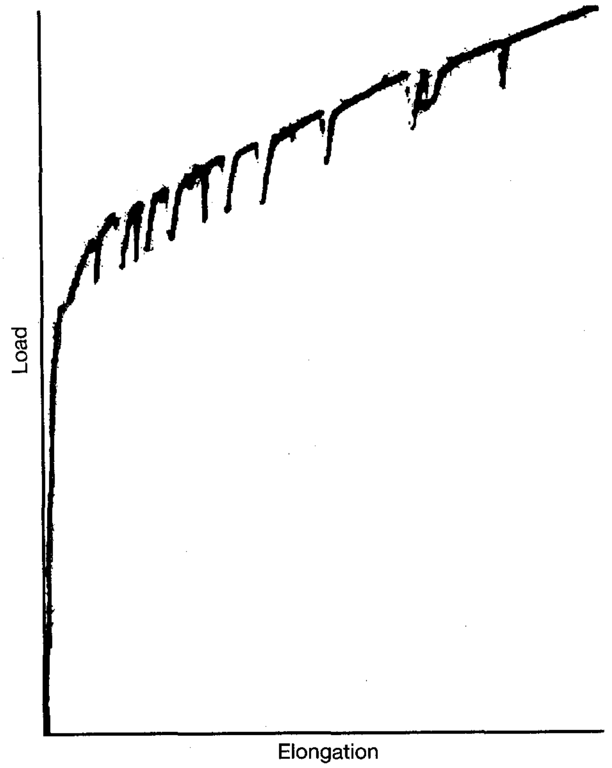

Figure 3 The original load–elongation curve for composite specimen no. 38. The maximum load is 245 N, the maximum elongation is 1.37 mm.

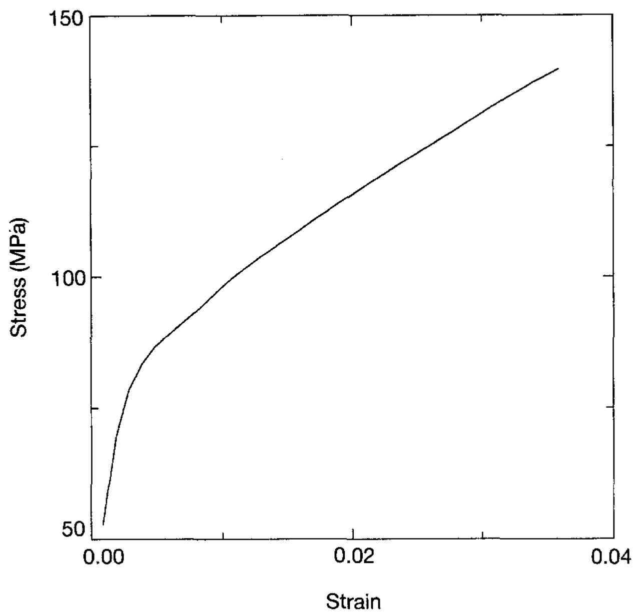

Figure 4 The stress–strain curve of the copper matrix (averaged after three tests).

For the same configurations the Cauchy problem was solved for various sets of the input parameters, $\bar{\sigma}$, β, τ, and fixed values of $\lambda = 4\,\text{mm}^{-1}$. Also, for a particular specimen, the values of $p_{\min}$ and $p_{\max}$ were chosen and then fixed. The least squares procedure was then used to get the best fit of the calculated curves $k(p)$ to the experimental points (Figs 5–7) and to choose on this basis proper values of these parameters characterizing a given fibre. The parameters are shown in the Table I.

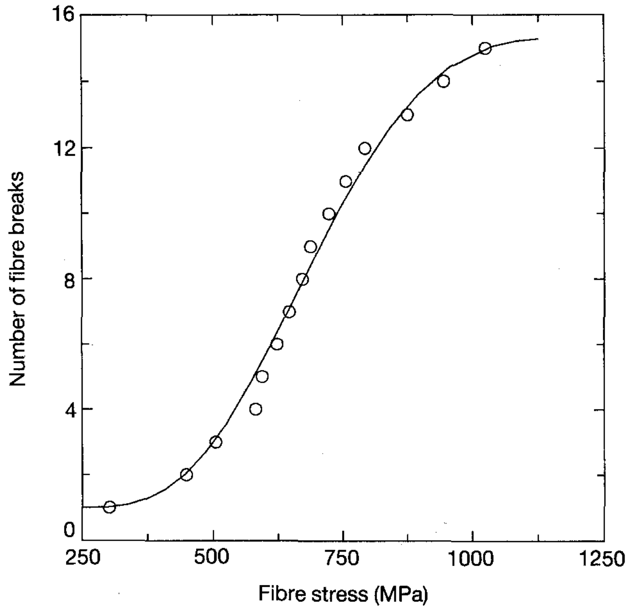

Figure 5 The experimental and calculated dependencies of the number of fibre breaks upon fibre stress. The fibre strength parameters providing the best fit are shown in the Table I. Composite specimen no. 38 (oxide fibre/copper matrix).

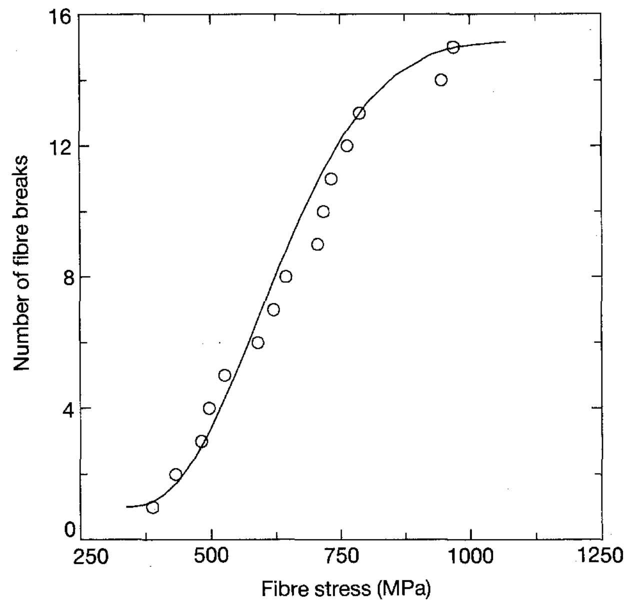

Figure 6 The experimental and calculated dependencies of the number of fibre breaks upon fibre stress. The fibre strength parameters providing the best fit are shown in the Table I. Composite specimen no. 39 (oxide fibre/copper matrix).

5. Conclusions

An attempt at a strict formulation of the problem involved in the description of the fibre fragmentation test is proposed, and an approximate solution of the problem is analysed to obtain the statistical fibre strength characteristics.

The procedure is developed to interpret the results of a tensile test of a single-filament composite. It is shown that a complete description of fibre strength requires the recording in the experiment a stress–strain curve with fibre break points on it and the stress–strain curve of the matrix. The technique has

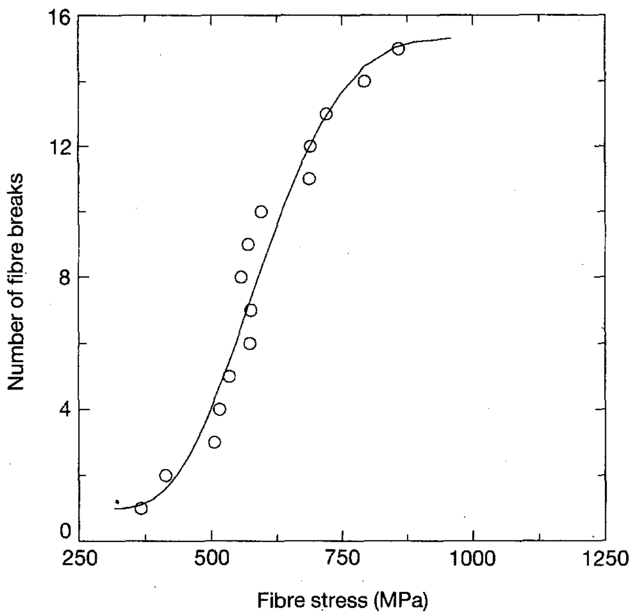

Figure 7 The experimental and calculated dependencies of the number of fibre breaks upon fibre stress. The fibre strength parameters providing the best fit are shown in the Table I. Composite specimen no. 48 (oxide fibre/copper matrix).

TABLE I Strength characteristics of fibres

Composite no.	β	$\bar{\sigma}$ (MPa)	τ^* (MPa)	p_{min} (MPa)	p_{max} (MPa)
38	4.48	1061	42.8	141	1266
48	4.20	1001	38.1	184	1195
49	4.55	775	40.1	173	1039

been applied to a study of oxide composite fibres developed for a high temperature use, and appeared to give a sound outcome.

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